sense that

$$Hp_a(x) = E_a p_a(x),\ Hq_a(x) = E_a q_a(x). \qquad (10)$$

where the twofold degeneracy is evident. The '*supercharges*' Q and $\bar{Q}$ such that

$$H = Q\bar{Q} + \bar{Q}Q + \bar{H} \qquad (11)$$

where $\bar{H}$ is an element of the kernel of $P_{2j}(x)$, are

$$Qp_a = 0,\ Qq_a = \sqrt{E_a}p_a,$$
$$\bar{Q}p_a = \sqrt{E_a}q_a,\ \bar{Q}q_a = 0,$$

in agreement with (10). Some more specific examples can be found in [10].

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SUPERSYMMETRY METHODS, in quantum physics — The idea of **supersymmetry** was originally introduced in relativistic *quantum field theory* as a possible generalization of *Poincaré symmetry*. In 1976 Nicolai [1] suggested a similar generalization for nonrelativistic quantum mechanics, the so-called **supersymmetric quantum mechanics**. With the one-dimensional quantum model introduced by Witten [2] in 1981, **supersymmetry** became a major tool in quantum mechanics and mathematical, statistical, and condensed matter physics. **Supersymmetry** is also a very successful concept in atomic and nuclear physics. An underlying supersymmetry of a given quantum mechanical system can be utilized to analyze the properties of this system in an elegant and effective way. It is even possible to obtain exact results thanks to the algebraic structure implied by supersymmetry.

Despite the fact that **supersymmetric quantum mechanics** is indeed the $(0+1)$-dimensional limit of SUSY *quantum field theory* it is rather independent of the latter. SUSY in SUSY quantum mechanics is not the original supersymmetry relating bosons and fermions. The *supercharges* of SUSY quantum mechanics do not generate transformations between bosons and fermions. They generate transformations between two orthogonal eigenstates of a given Hamiltonian with the same degenerate eigenvalue.

A quantum mechanical system which is characterized by a Hamiltonian H and a set of self-adjoint *supercharges* $\{Q_1, \ldots, Q_N\}$ operating on a common *Hilbert space* is called supersymmetric if the following **anticommutation relation** is valid for all i and j:

$$\{Q_i, Q_j\} = H\,\delta_{ij}. \qquad (1)$$

As an example for the simplest case N = 1 let us consider the supercharge

$$Q_1 := \frac{1}{\sqrt{4m}}\left(\vec{p} - \frac{e}{c}\vec{A}\right)\cdot\vec{\sigma} \qquad (2)$$

acting on $L^2(R^3) \otimes C^2$. If we identify the parameters m and e with the mass and the charge of a spin-$\frac{1}{2}$ particle in the presence of an external magnetic field $\vec{B} := \vec{\nabla} \times \vec{A}$ characterized by a vector potential $\vec{A}$ we realize that the corresponding SUSY Hamiltonian

$$H := \{Q_1, Q_1\} = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 - \frac{e\hbar}{2mc}\vec{B}\cdot\vec{\sigma} \qquad (3)$$

coincides with the well-known *Pauli Hamiltonian* for this particle having a *gyromagnetic factor* $g = 2$. It is an amusing and interesting observation that supersymmetry suggests $g = 2$, a fact which usually follows from the relativistic covariant *Dirac Hamiltonian* only. As an aside let us note that supersymmetry is a rather natural symmetry in relativistic quantum mechanics. For example, the above supercharge Q_1 also appears in the associated *Dirac Hamiltonian* of this system in a natural way [3,4].

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SUPERSYMMETRY METHODS, in statistical physics — Methods which transform problems in statistical physics to problems in supersymmetric *field theory*, or vice-versa. These techniques usually involve the replacement of random variables (in a functional integral or differential equation) by *Grassmann variables*, with a resulting *effective action* that exhibits some form of supersymmetry. One can then exploit both the techniques of standard *field theory* and the supersymmetry itself, so that the original problem is simplified or made more tractable. In some cases the transformation also provides deeper insights. The original variables may represent disorder or other random perturbations of the system. Let us consider several examples [1–4].